

Reacting Radial Flow of a Power-law Fluid with Variable Viscosity through a Porous Medium

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Abstract— In this work, we are focusing on the reacting radial flow of a power-law fluid with variable viscosity through a porous medium. We considered the governing equations with reaction term and variable viscosity. We employed Galerkin weighted residual method to analyze the problem and we report the effects of various physical parameters involved in the model graphically.

Index Terms— Power-law fluid, Porous medium Reacting term, Radial flow.

1 INTRODUCTION

The unsteady and steady flow of Newtonian and non-Newtonian fluids in porous media in which the main driving force is gravity has attracted the attention of many scientists in the recent times. This is due to its large area of applications in engineering practices, particularly in applied geophysics, geology, groundwater flow, food technology, filtration processes, enhanced oil recovery, oil reservoir engineering and oil recovery processes, to mention but just a few.

Reservoir engineers deal with the flow of oil, water and gas in petroleum reservoirs. Although, the non-Newtonian behavior of many fluids has been recognized for a long time, the science of rheology is still in its infancy in many respects. As such, new phenomena are being discovered on a constant basis with new theories propounded. Advancement in computational techniques are making possible much more detailed analyses of complex flow and complicated simulation of the structural and molecular behavior that give rise to non-Newtonian behaviors. Engineers, Chemists, Physicists and Mathematicians are actively pursuing research in rheology. Flows through po-

rous media are very prevalent in nature, and therefore studies of flow through a porous medium has attracted the interest of many scientists who are directly or indirectly involved in various engineering applications. In some recent researches the variation of viscosity and chemical reaction parameters were considered.

Kumar and Prasad [1] considered MHD pulsatile flow through a porous medium. Analytical solution was employed in solving the system of flow. Their result shows that an increase in the permeability parameter and Hartmann number leads to a decrease in the steady state velocity. Vajraveh et al [2] examined fluid flow and heat transfer over a permeable stretching cylinder. A numerical method involving second order finite difference scheme known as Keller Box method was employed to investigate the velocity and temperature distribution of the flow system. Their result shows that increasing values of the fluid viscosity parameter is to enhance the temperature. This is due to the fact that an increase in the fluid viscosity parameter results in an increase in the thermal boundary layer thickness. The effects of variable viscosity, viscous dissipation and chemical reaction on heat and

mass transfer flow of MHD micropolar fluid along a permeable stretching sheet was examined by Salem [3]. A numerical method involving Runge-Kuta fourth order method and shooting technique were employed to investigate the velocity and temperature distribution of the flow system. The results show that as Prandtl number and viscosity parameter increases the velocity profile and the temperature profile decreases.

The effect of variable viscosity and thermal conductivity of micro polar fluid in a porous channel in the presence of magnetic field was studied by Gitima [4]. A numerical method involving Runge-Kuta fourth order method was employed to investigate the velocity and temperature distribution of the flow system. The results show that the velocity and temperature of the fluid increases as Darcy number, thermal conductivity variation parameter and magnetic field parameter increases. Cortell [5] investigated on unsteady gravity flows of a power-law fluid through a porous medium. He analyzed the flow in two direction, one side both thinning and thickening of the fluids and on the other hand, two different types of solutions, for the case of a gravity flow generated by the injection of a power-law fluid at the well into an empty reservoir of an infinite extent. He employed shooting method to analyze the flow model. The result shows that as power-law index increases the velocity profile decreases. Ogunsola and Ayeni [6] considered the effects of temperature distribution of an Arrhenius reacting unsteady flow through a porous medium with variable permeability. A numerical method involving shooting method was employed to investigate the velocity

and temperature distribution of the flow system. Their result shows that as Frank-Kamenetskii parameter increases the fluid velocity and temperature increases. Motivated by these facts, the present work has been undertaken in order to analyze a reacting radial flow of a power-law fluid with variable viscosity through a porous medium.

2.0: MATHEMATICAL FORMULATION AND METHOD OF SOLUTION

Radial Flow in Porous media

The governing equations are conservation of mass, momentum and energy.

$$\frac{1}{r} \frac{\partial(rhu)}{\partial r} = -\Phi \frac{\partial h}{\partial t} \quad (2.1)$$

$$u = \left(\frac{K\rho}{\mu_{ef}} \right)^{1/n} \left(\frac{\partial h}{\partial r} \right)^{1/n} \quad (2.2)$$

$$\rho C_p \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(k_0 e^{-\gamma T} r \frac{\partial T}{\partial r} \right) + \mu \left(\frac{\partial u}{\partial r} \right)^2 + Q(T) \quad (2.3)$$

The appropriate initial and boundary conditions are

$$\begin{aligned} h(r,0) &= h_0, h(0,t) = h_1, h(\infty,t) = 0 \\ T(r,0) &= T_0, T(0,t) = T_1, T(\infty,t) = T_0 \quad t > 0 \end{aligned} \quad (2.4)$$

where

r is the distance of the given point of the reservoir from the axis of symmetry, k - Thermal conductivity, ρ - Density, C_p

-Specific heat at constant pressure, μ -Dynamic viscosity,

$\mu(\partial u / \partial r)^2$ is the viscous heating term, V_r -Component of velocity

in the radial direction, Q - Heat release per units mass, E -

Activating energy, R - Universal gas constant, μ_{ef} -Effective

viscosity, n -Dimensionless Power-law index, e^T -Thermal ex-

pansion, k_0 - The thermal conductivity of the fluid, γ -Thermal

expansion exponent, T_0 -Initial temperature and it is the refer-

ence temperature, T -Temperature within the boundary layer,

$T_1, T_2, \dots, T_\infty$ -Temperature at the plate,

$H = p - \rho g z = \rho g h$, $p = \rho g h$, η -Apparent viscosity, i.e.

Similarity variable parameter, f -is a dimensionless stream func-

tion, m_0 or k_1 - Flow consistency index, θ -Dimensionless

temperature, A - pre-exponential factor, RT - average kinetic

energy.

Reynolds's model : We assume the fluid viscosity to vary as an exponential function of temperature in the non-dimensional form, following Gitima [4]

$$\mu(T) = \mu_0 e^{-M(T-T_0)} \quad (2.5)$$

where μ_0 is the constant value of the coefficient of viscosity far away from the surface, M is the variable viscosity parameter.

Substituting Equation (2.5) into (2.1) we obtain

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\left(e^{M_1 \theta / n} \right) \left(\frac{\partial h}{\partial r} \right)^{\frac{1}{n}} h r \right) = a^2 \frac{\partial h}{\partial t} \quad (2.6)$$

$$\text{where } M_1 = M(T_1 - T_0), a^2 = \Phi \left(\frac{\mu_0}{K \rho} \right)^{\frac{1}{n}} \quad (2.7)$$

We seek similarity solution of the form

$$h(r, t) = h_3 t^\alpha f(\eta); \eta = r t^\beta, \theta(r, t) = t^\lambda \phi(\eta) = t^\lambda g(\eta), \\ h_1 = h_3 t^\alpha \quad (2.8)$$

where $\alpha, \beta \in \Re$ and f is a function of η only

Substitute Eq. (2.8) into (2.6) we have

$$\Rightarrow e^{\frac{M_1 \phi}{n}} \left(f'(f')^{\frac{1}{n}} + f \phi' (f')^{\frac{1}{n}} \frac{M_1}{n} + f \frac{d}{d\eta} (f')^{\frac{1}{n}} + f (f')^{\frac{1}{n}} \right) \\ = a^2 (\alpha f + \beta \eta f') \quad (2.9)$$

$$f(0) = 1, f(\infty) = 0, \phi(0) = 1, \phi(\infty) = 0 \quad (2.10)$$

We shall now proceed to seek a numerical solution of Equation (2.9) together with the boundary conditions (2.10) using Galerkin-Weighted Residual Method as follows:

$$\text{let } f = \sum_{i=0}^2 A_i e^{(-i/4)\eta} \quad (2.11)$$

A maple pseudo code was used to solve Equation (2.10)

The result are presented in Figures 2.1-2.2

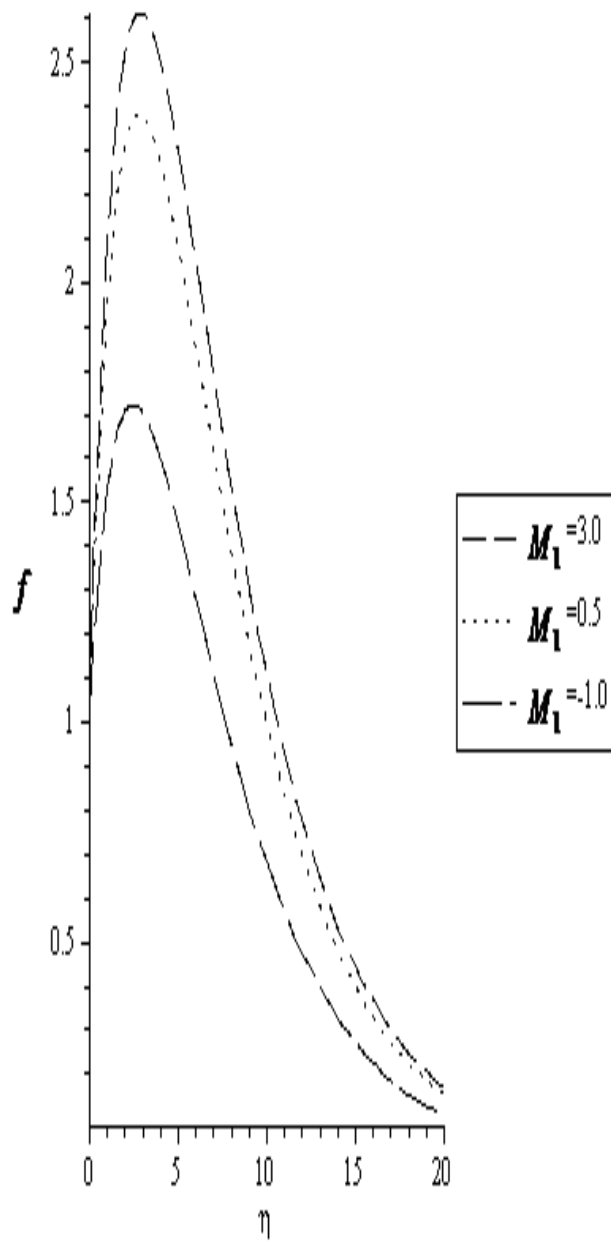


Figure 2.1: Graph of the velocity function f for various values M_1 when $\alpha = 0.5, a = \beta = n \geq 0.1$.

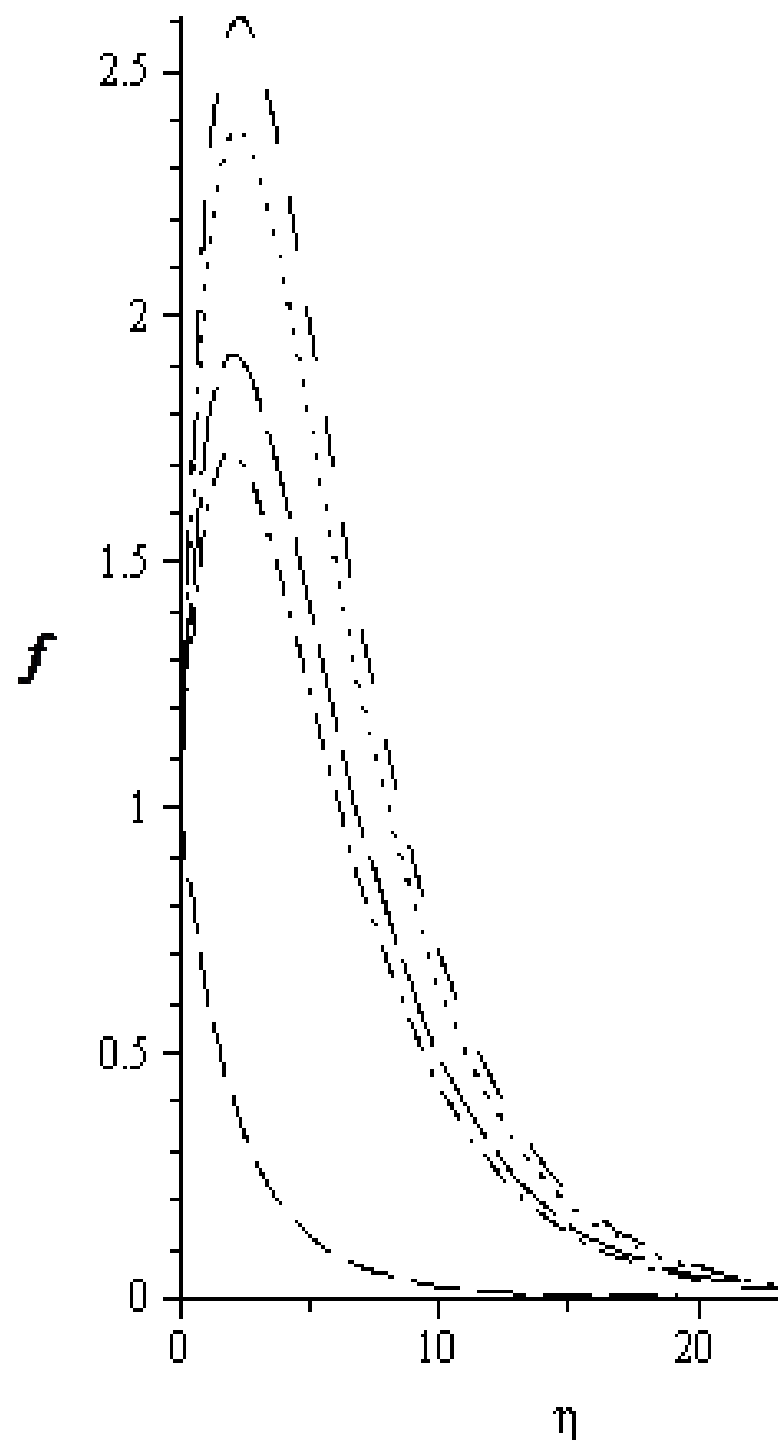


Figure 2.2: Graph of the velocity function f for various values power-law index when $\alpha = 0.5, a = \beta = M_1 = \text{Pr} = \text{Br} = \text{Re} = 1.0$.

We further consider the energy equation of an Arrhenius re-

acting flow through a porous medium with variable thermal

$$\theta(r, t) = t^\lambda \phi(\eta), \eta = rt^\beta$$

conductivity as follows:

$$\frac{1}{r} \frac{\partial(rhu)}{\partial r} = -\Phi \frac{\partial h}{\partial t} \quad (2.12)$$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(k_0 e^{-\gamma T} r \frac{\partial T}{\partial r} \right) + \mu \left(\frac{\partial u}{\partial r} \right)^2 + Q(T) \quad (2.13)$$

The appropriate initial and boundary conditions are

$$h(r, \infty) = h_0, h(0, t) = h_1, h(\infty, t) = 0$$

$$T(r, 0) = T_0, T(0, t) = T_1, T(\infty, t) = T_1 \quad t > 0 \quad (2.14)$$

Following [6] we consider reaction term of the form

$$Q(T) = Q_0 A t^{-a} e^{-E/RT} \quad (2.15)$$

Substituting Eq.(2.15) into (2.13) we have

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(k(T) r \frac{\partial T}{\partial r} \right) + \mu \left(\frac{\partial u}{\partial r} \right)^2 + Q_0 A t^{-a} e^{-E/RT} \quad (2.16)$$

We introduce the following non-dimensional variables

$$t' = \frac{t}{t_0}, \theta(r, t) = \frac{T - T_0}{T_1 - T_0}, r' = \frac{r}{R}, h' = \frac{h}{h_0},$$

$$t = t' t_0, T = \theta(r, t) [(T_1 - T_0) + T_0], r = r' R$$

$$\varepsilon = \frac{RT_0}{E}, \theta = \frac{E(T - T_0)}{RT_0^2}, T = \frac{\theta RT_0^2}{E} + T_0, T - T_0 = \varepsilon T_0 \theta, T = \varepsilon T_0 \theta + T_0 = T_0 (\varepsilon \theta + 1)$$

$$(2.17)$$

substituting the dimensionless variable Eq. (2.17) into (2.16) we

obtain

$$\beta \eta \phi' = e^{-s\phi} \left[-k_0 \frac{Br}{Pe} \gamma (\phi')^2 + \frac{1}{Re} (\eta^{-1} \phi' + \phi'') \right] + p_0 \left[\frac{d}{d\eta} (f')^{\frac{1}{n}} \right]^2 \quad (2.18)$$

$$\text{where } p_0 = \frac{\mu t_0}{T_1 - T_0} \left(\frac{K\rho}{\mu_{ef}} \left(\frac{h_0}{R} \right) \right)^{\frac{2}{n}}, R_6 = e^{-\gamma_1 T_0} \frac{t_0}{R^2},$$

$$Br = \frac{u_0^2 \mu R_5}{k_0 (T_1 - T_0)},$$

$$Pr = \frac{\mu \rho c_p}{R_6 k_0}, Re = \frac{l_0 u_0 \rho}{\mu}, R_5 = \frac{e^{-\gamma_1 T_0} t_0 \rho (T_1 - T_0)^2}{l_0 R^2 \mu u_0},$$

$$\psi = \frac{t_0 t_0^{-a}}{T_1 - T_0} \frac{EQ_0 A}{\rho C_p (RT_0^2 + T_0 E)} e^{-E/RT_0},$$

ψ being Frank-kamenetskii parameter for the system.

$$(2.19)$$

$$\therefore \phi(0) = 1, \phi(\infty) = 0$$

$$(2.20)$$

Case1: when $\varepsilon = 0$

We now proceed to solve Equations (2.19) and (2.9) subject to

(2.10) & (2.20) numerically using Galerkin-Weighted Residual

Method as follows:

$$\text{let } f = \sum_{i=0}^2 A_i e^{(-i/5)\eta}, \phi = \sum_{i=0}^2 B_i e^{(-i/4)\eta}$$

$$(2.21)$$

A maple pseudo code was used to solve problem (2.21)

The result are presented in Figures 2.3-2.5

Figure 2.3: Graph of the velocity function f for various values

power-law index n

when $\beta = \text{Re} = \text{Pr} = Br = 0.05$, $k_0 = 2.0$, $\alpha = 2.5$.

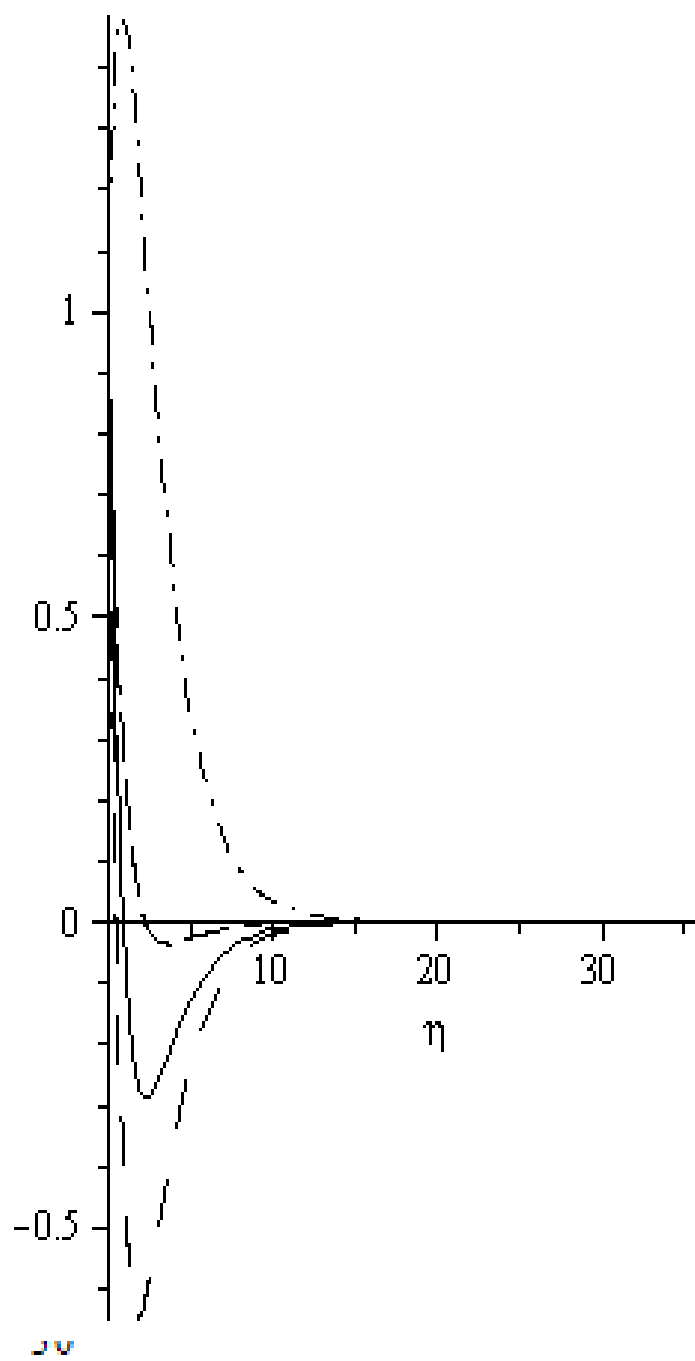
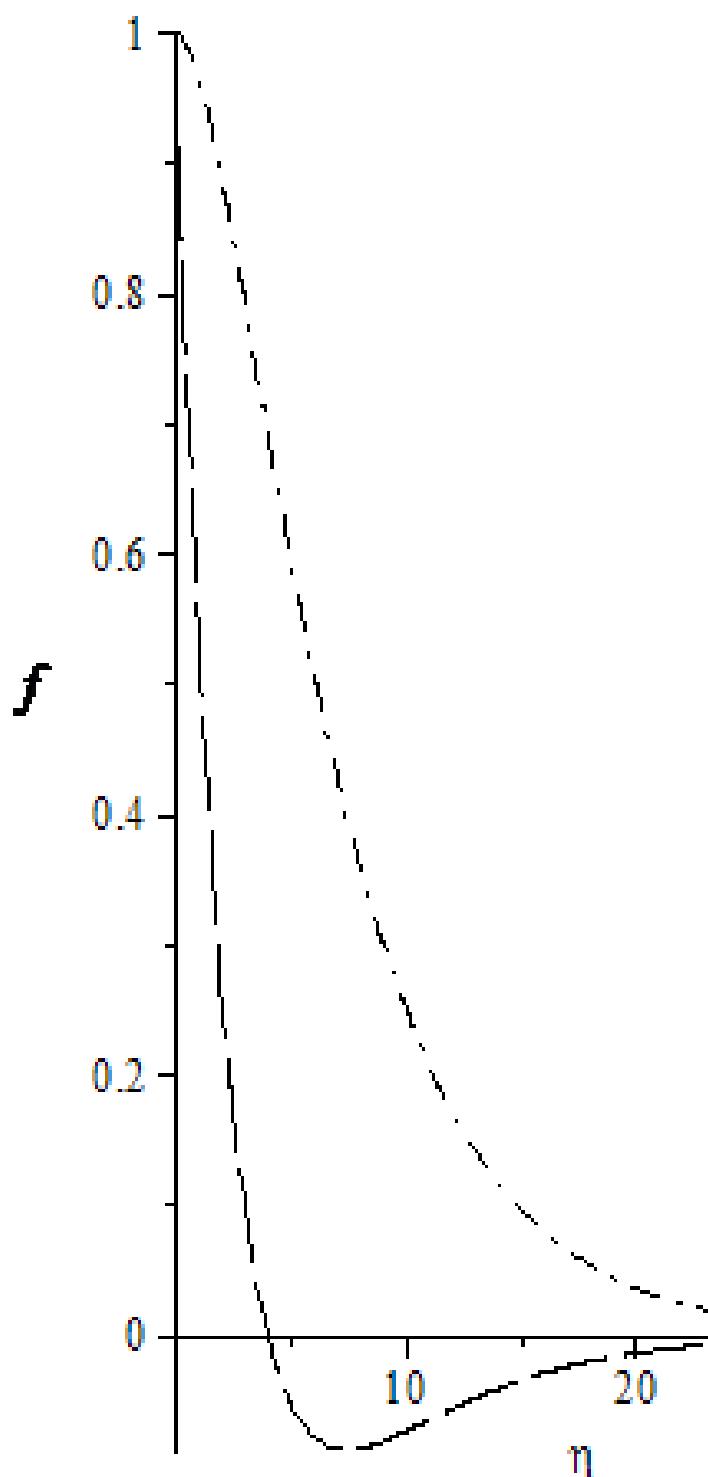


Figure 2.4: Graph of the temperature function ϕ for various

values

ψ , Frank – Kamenetskii parameter

when $\beta = \text{Re} = \text{Pr} = \text{Br} = 1.5$, $k_0 = 1.0$, $n = 3.0$, $\alpha = 0.5$

Case2: when $\varepsilon = 0.25$

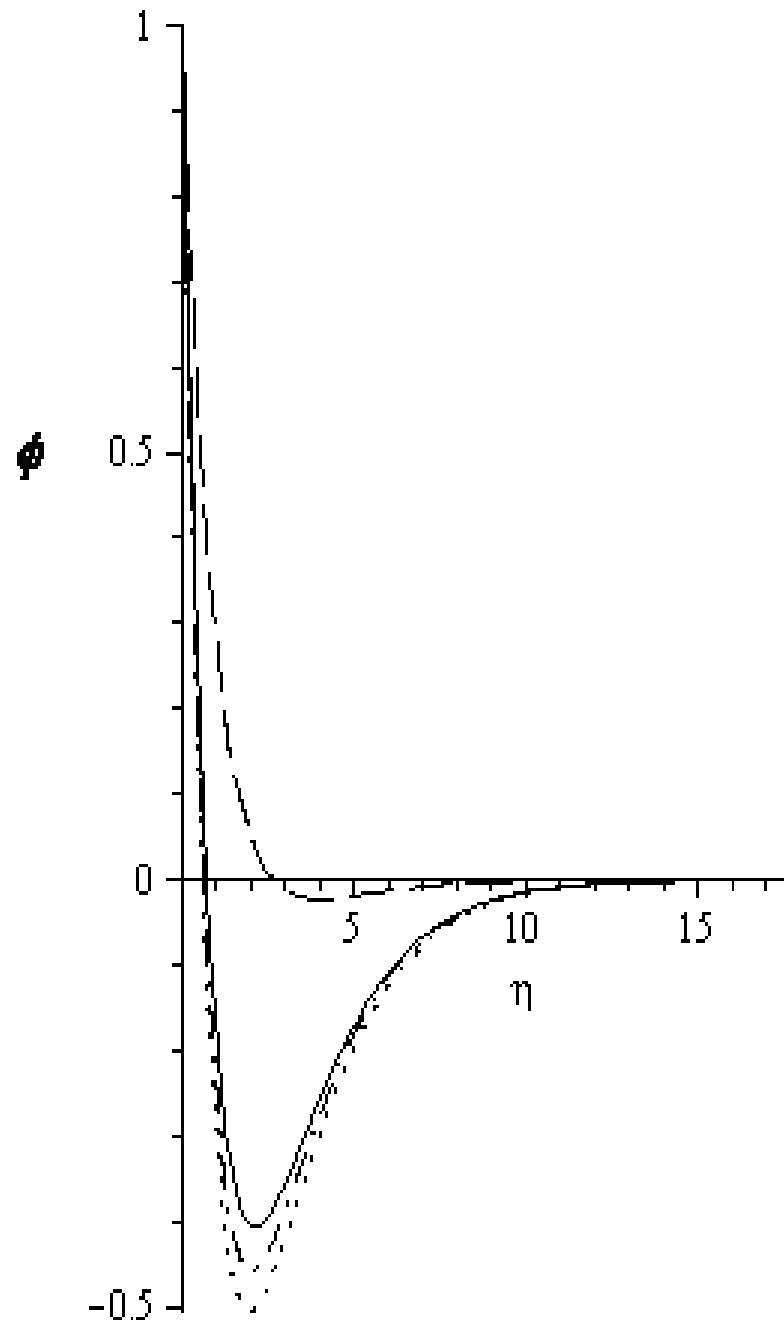
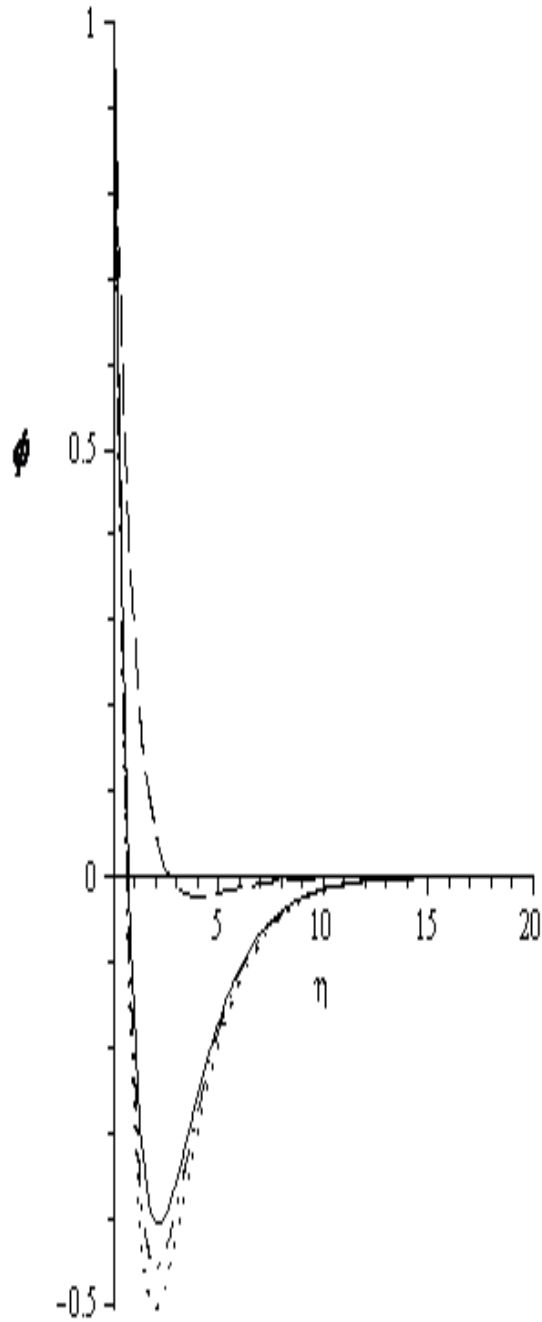


Figure 2.5: Graph of the temperature function ϕ for various values of ψ , Frank – Kamenetskii parameter when $\beta = \text{Br} = \text{Pr} = \text{Re} = 1.2$, $k_0 = 0.50$, $n = 2.0$, $\alpha = 0.5$

3.0: Discussion of Results and Conclusion

Discussion of Results

We have considered a suitable model of unsteady variable thermal conductivity gravity flow of a power-law fluid with viscous dissipation through a porous medium. The result from Figure 2.1 shows that the velocity profile decreases with a decrease in M_1 variable viscosity parameter. From Figure 2.2 the result shows that the velocity profile decreases monotonically as parameters n increases. The results from Figures 2.4 and 2.5 show that the fluid temperature decreases with increase in each of Brinkman number, Prandtl number, Reynolds number, viscous dissipation parameter, Frank-Kamenetskii parameter and thermal conductivity parameter. The result from Figure 2.4 shows that the temperature profile decreases for $\varepsilon = 0$, $n = 0.01$ and $\psi \leq 0.4$. It is observed from Figure 2.5 that the fluid temperature distribution decreases for $\psi \leq 0.5$ and $n < 0$. More so, we noticed that an increase in viscous dissipation parameter and *Frank – Kamenetskii* parameter have a considerable effect on both the velocity profile as well as the fluid temperature. On the other hand, from numerical calculations we also see that the parameter $n, s, Pr, Re, p_0, \beta, \varepsilon, Br$ and ψ affects the flow characteristics significantly.

Conclusion

A set of non-linear coupled differential equations governing the fluid temperature is solved analytically and numerically

for various parameters. It is noted that the influence of variable viscosity parameter and Frank-Kamenetskii parameter on the flow system is to increase the fluid temperature.

It can be concluded that the increase physical parameters i.e. Reynolds number, Prandtl number, Brinkman number and Peclet number; thermal conductivity parameter, Frank-Kamenetskii parameter, viscous dissipation parameter, and variable viscosity parameter leads to a corresponding decrease in the viscosity of the fluid. This will be of great importance for the field engineers in various processes of oil recovery.

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