Int. J. Pure Appl. Sci. Technol., 22(1) (2014), pp. 1-8

International Journal of Pure and Applied Sciences and Technology ISSN 2229 - 6107 Available online at www.ijopaasat.in

Research Paper Effect of Variable Viscosity on Third Grade Fluid Flow over a Radiative Surface with Arrhenius Reaction

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(Received: 22-3-14; Accepted: 27-4-14)

Abstract: In this work, effects of variable viscosity and Arrhenius reaction on the third grade fluid over a radiative surface through a porous medium is considered. The governing partial differential equations were transformed into ordinary differential equations in terms of suitable similarity variable. We employed Galerkin weighted residual method to solve the resulting non-linear equations. The effects of variable viscosity parameter, Frank-Kamenetskii parameter, Brinkman number, Reynolds number, Prandtl number and Darcy number on the system of flow and the results were reported graphically.

Keywords: Third grade fluid, weighted residual method, Arrhenius reaction, radiative surface, and porous medium.

Introduction

Non-Newtonian fluids have received much attention than Newtonian fluids in the recent years due to its practical importance, rapid development of modern industrial materials and technological applications. It has given insight in the reservoir engineering, heat exchange between soil and atmosphere, flow of moisture through porous industrial materials, heat exchangers with fluid beds, preheating coal-water mixture, ceramic processing, catalytic reactors, polymer solution, molten plastics, oil recovery, to mention but just a few applications. Some materials behave non-Newtonianly namely mud, pasta, cheese, personal care product, asphalt, ice cream, oils and the host of others. The rheological characteristics of polymer melts and solutions together with the attributes of some polymeric substances, which has lead to the recent world-wide processing of polymer. The rheological behaviour of all the non-Newtonian fluids cannot be determined by a single constitutive equation. Heat transfer problem of third grade fluids has been studied by several authors: Makinde and Aziz [1] considered the boundary layer flow of viscous nanofluid bounded by a stretching sheet. Mukhopadhyay [2] examined the boundary layer flow and heat transfer towards an exponentially stretching porous sheet embedded in a porous medium. Krishnendu et al [3] considered similarity solution of mixed convective boundary layer slip flow over a vertical plate. Fosdick and Rajagopal [4] performed a complete thermodynamic analysis of constitutive equations for the third grade fluid involving heat transfer process. Massoudi and Christie [5] analyzed numerically the flow of a third grade fluid in a pipe without heat source where the shear viscosity was assumed to be temperature dependent. Olajuwon [6] examined the flow and natural convection heat transfer in a power-law fluid past a vertical plate with heat generation. Yurusoy et al [7] examined entropy analysis for third grade fluid flow with Vogel's models of viscosity in annular pipe. Nadeem et al [8] analytically considered the effects of partial slip on a fourth grade fluid with variable viscosity and Makinde [9] employed Hermite-Pade approximations to evaluate thermal radiation effect of inherent irreversibility in a variable viscosity channel flow. Massoudi and Christie [10] studied the effects of variable viscosity and viscous dissipation on the flow of third grade fluid in a pipe. Nadeem et al [11] studied analytical solutions for pipe flow of a fourth grade fluid with Reynolds and Vogel's models of viscosities. Lazarus [12] examined the effects of variable viscosity on the velocity fluid and temperature fluid using semi-implicit finite difference scheme of Laminar flow in a channel filled with saturated porous media. The results show that the velocity fluid and temperature fluid increases as variable viscosity parameter increases. Motivated by the work of Hayat et al [13] they considered the effect of joule heating and thermal radiation in flow of third grade fluid over a radiative surface.

Following Szeri and Rajagopal [4] an incompressible, homogeneous fluid of third grade is characterized by Cauchy stress τ of the following form:

$$\tau = -pI + \mu(T)A_1 + \alpha_1(T)A_2 + \alpha_2(T)A_1^2 + \beta_1(T)A_3 + \beta_2(T)[A_1A_2 + A_2A_1] + \beta_3(T)(trA_1^2)A_1.$$
(1)

where -pI denote the indeterminate part of the stress due to the constraint of incompressibility $\mu(T)$ is the coefficient of viscosity and $\alpha_1(T), \alpha_2(T)$ are material moduli, usually referred to as normal stress coefficients. The kinematic tensors A_1, A_2 are defined by [2] through

$$A_1 = (grad v) + (grad v)^{\tau}$$
⁽²⁾

$$A_{n} = \frac{d}{dt}A_{n-1} + A_{n-1}(grad v) + (grad v)^{T}A_{n-1} \quad n = 2,3.$$
(3)

Here $\frac{d}{dt}$ denotes material time derivative and v is the velocity vector. The above model contains, as a special subclass, the classical linearly viscous model (the case when all the coefficients expect μ are set equal to zero). Flow of a thermodynamically compatible fluid of third grade is given as:

$$v = u(y) i \tag{4}$$

where i denotes the unit vector in the x-coordinate direction, the direction that is chosen parallel the external pressure gradients.

In the absence of body forces, the balance of linear momentum

$$div \,\tau + \rho b - \frac{\mu_{ef} v}{K} = \rho \frac{dv}{dt}$$
 (Equation of Motion) (5)

where K is the porous medium permeability

$$\rho \frac{de}{dt} = \tau L - div \, q + \frac{\mu_{ef} v^2}{K} + \rho r + Q(T) \tag{6}$$

Here *e* denotes the internal energy, *L* is the velocity gradient, *r* is the radiant energy, both per unit mass and Q(T) denotes the reacting term.

Governing Equations and Method of Solution

Following Tawaser et al [13], we consider the unsteady two-dimensional boundary layer flow of a magneto hydrodynamic (MHD) through a porous medium. The basic governing equations are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left(\mu(T) \frac{\partial u}{\partial y} \right) + \frac{\alpha_1}{\rho} \left[\frac{\partial^3 u}{\partial t \partial y^2} + u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + 3 \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y^2} \right]$$
$$+ 2 \frac{\alpha_2}{\rho} \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y^2} + 6 \frac{\beta_3}{\rho} \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} + \frac{\sigma}{\rho} \left(E_0 B_0 - B_0^2 u \right) + \mu_{ef} \frac{u}{K}$$

$$k\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) + \mu(T)\left(\frac{\partial u}{\partial y}\right)^{2} + \alpha_{1}\left[\frac{\partial u}{\partial y}\frac{\partial^{2}u}{\partial t\partial y} + u\frac{\partial u}{\partial y}\frac{\partial^{2}u}{\partial x\partial y} + v\frac{\partial u}{\partial y}\frac{\partial^{2}u}{\partial y}\right] + 2\beta_{3}\left(\frac{\partial u}{\partial y}\right)^{4} + \sigma(uB_{0} - E_{0})^{2} + \mu_{ef}\frac{u^{2}}{K} + Q(T) = \rho c_{p}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)$$
(9)

The appropriate initial and boundary condition are as follows

$$u(x,0) = U_w, v(x,0) = V_w, T(x,o) = T_w, \text{ as } y = 0, T \to T_w \text{ as } y \to \infty$$

$$(10)$$

where σ^* is the Stefan-Boltzmann constant, k_1 is the mean absorption coefficient, Q is the heat release per units mass, $V_w < 0$ is the injection, $V_w > 0$ is the suction, $V_w = -\frac{v_0}{(1-ct)^{1/2}}$,

$$U_{w} = -\frac{ax}{(1-ct)}, T_{w}(x,t) = T_{\infty} + T_{0}\frac{ax}{2v(1-ct)^{2}}, a, b \text{ are constants}, E \text{ is the activating energy}, R \text{ is}$$

the universal gas constant, θ -is the dimensionless temperature, k is the permeability of the porous media, K is the thermal conductivity, ρ is the density, C_p is the specific heat at constant pressure, μ is the dynamic viscosity, $\mu \left(\frac{\partial u}{\partial r}\right)^2$ is the viscous heating effect, direction, ψ is the Frank-Kamenetskii parameter, η is the similarity variable, μ_{ef} is the effective viscosity, e^T is the thermal expansion, T_0

(8)

is the fluid initial temperature or wall temperature, T_r is the reference temperature, T is the absolute temperature within the boundary layer, T_1 , T_2 ,.... T_{∞} - Temperature at the plate, $\alpha_1, \alpha_2, \beta_3$ are the fluid parameters, u is the dimensionless velocity in the x direction, Ψ is the stream function, y is the dimensionless velocity in the y direction , T_{∞} is the free stream temperature.

From Equations (8) and (9) we seek Reynolds model of the form

$$\mu(T) = \mu_0 e^{-M\theta} \tag{11}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left(\mu_0 e^{-M\theta} \frac{\partial u}{\partial y} \right) + \frac{\alpha_1}{\rho} \left[\frac{\partial^3 u}{\partial t \partial y^2} + u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + 3 \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y^2} \right] + 2 \frac{\alpha_2}{\rho} \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y^2} + 6 \frac{\beta_3}{\rho} \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} + \frac{\sigma}{\rho} \left(E_0 B_0 - B_0^2 u \right) + \mu_{ef} \frac{u}{K}$$

$$(12)$$

$$k\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) + \mu_{0}e^{-M\theta}\left(\frac{\partial u}{\partial y}\right)^{2} + \alpha_{1}\left[\frac{\partial u}{\partial y}\frac{\partial^{2}u}{\partial t\partial y} + u\frac{\partial u}{\partial y}\frac{\partial^{2}u}{\partial x\partial y} + v\frac{\partial u}{\partial y}\frac{\partial^{2}u}{\partial y}\right] + 2\beta_{3}\left(\frac{\partial u}{\partial y}\right)^{4} + \sigma\left(uB_{0} - E_{0}\right)^{2} - \left(\frac{\partial q_{r}}{\partial y}\right)^{2} + \mu_{ef}\frac{u^{2}}{K} + QC_{0}Ae^{-E_{RT}}$$
$$= \rho c_{p}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)$$
(13)

We introduce the following variables and parameters

$$\Psi = \sqrt{U_w v x} f(\eta), T = T_{\infty} + (T_w - T_{\infty}) \theta(\eta) \text{ and } \eta = y \sqrt{U_w / x v} y, u = \frac{\partial \Psi}{\partial y}, v = -\frac{\partial \Psi}{\partial x},$$

$$\psi = \frac{EQC_0A}{\left(RT_0^2 + T_0E\right)}e^{\frac{-E}{RT_0}}, R_d = \frac{4\sigma^*T_\infty^3}{k^*K}, y' = \frac{y}{lo}, u' = \frac{u}{u_0}, \theta = \frac{T - T_{wr}}{T_1 - T_w}, Da = \frac{k}{Ulo^2}.$$
 (14)

Substituting Equation (14) into Equations (12) and (13), we obtain

$$\frac{d}{d\eta} \left(e^{-M\theta} f'' \right) - S\left\{ f' + 1/2 \eta f'' \right\} + \alpha_0 \left[2 f f''' - f f^{(iv)} + 3 f''^2 + S\left\{ 2 f''' + \frac{1}{2} \eta f^{(iv)} + \right\} \right]$$

$$+ 2\alpha_3 f''^2 + 6\beta \operatorname{Re} f''^2 f''' + m^2 \left(E_1 - f' \right) - \frac{u}{Da} = 0$$
(15)

$$\frac{1}{\Pr}\theta''\left(1+\frac{4}{3}R_d\right) + \left[f'\theta - f\theta' + \frac{S}{2}\left\{\eta\theta' + 4\theta\right\}\right] + Bre^{-M\theta}\left(f'\right)^2 + \alpha_3 Ec$$

$$\left[ff'''^2 + \frac{S}{2}\left\{3f''^2 + \eta f'f'''\right\} - ff'f''''\right] + 2\beta Ecf''^4 + m^2 Ec\left|f' - E_1\right|^2 + \frac{u^2}{Da} + \psi e^{\frac{\theta}{(1+\varepsilon\theta)}} = 0$$
(16)

Where

$$\psi = \frac{EQC_0A}{\left(RT_0^2 + T_0E\right)}e^{-E/RT_0}, R_d = \frac{4\sigma^*T_\infty^3}{k^*K}, \alpha_0 = \frac{\alpha_1a}{\mu_0(1-ct)}, \alpha_3 = \frac{\alpha_2a}{\mu_0(1-ct)}, \beta = \frac{\beta_3a^2}{\mu_0(1-ct)^2}, S = \frac{c}{a}.$$
(17)

The transformed boundary condition is as follows

$$f(0) = A, f'(0) = 1, f(\infty) = 1, f'(\infty) = 0, \theta(0) = 1, \theta(\infty) = 0$$
(18)

We proceed to solve Equations (15) and (16) subject to (18) numerically using Galerkin-Weighted Residual Method as follows:

Let
$$f = \sum_{i=0}^{2} A_i e^{v}, \theta = \sum_{i=0}^{2} B_i e^{\binom{1/4}{y}}$$
 (19)

The results are presented in Figures 1-5



Figure 1: Graph of the velocity function *f* for various values of Br = 0.5, $Pr = Re = A = S = \beta = 1.0$, Ec = 0.5, $\alpha_0 = \alpha_3 = m = E_1 = \psi = 0.1$.



Figure 2: Graph of the temperature function θ for various values of Br = 0.5, $Pr = Re = A = S = \beta = 1.0$, Ec = 0.5, $\alpha_0 = \alpha_3 = m = E_1 = \psi = 0.1$.



Figure 3: Graph of the velocity function f for various values of Br = 0.5, $Pr = Re = A = S = \beta = 1.0$, Ec = 0.5, $\alpha_0 = \alpha_3 = m = E_1 = \psi = 0.1$.



Figure 4: Graph of the temperature function θ for various values of Br = 0.5, $Pr = Re = A = S = \beta = 1.0$, Ec = 0.5, $\alpha_0 = \alpha_3 = m = E_1 = \psi = 0.1$.



Figure 5: Graph of the temperature function θ for various values of Br = 0.5, $Pr = Re = A = S = \beta = 1.0$, Ec = 0.5, $\alpha_0 = \alpha_3 = m = E_1 = \psi = 0.1$.

Discussion of Results/Conclusion

The study of third grade fluids is extremely important due to its wide variety of practical applications in processes such as filtration of polymer solutions and soil remediation through the removal of liquid pollutants. From Figures 1-5 the results show that the fluid velocity and temperature increases

increase in each parameters. It is noticed from Figures 3 & 5 that the temperature profile reach the maximum point as Darcy number *Frank* – *Kamenetskii* parameter increases. We observed from Figure 4 that the velocity profile has a minimum point as *Frank* – *Kamenetskii* parameter increases.

Conclusion

It is concluded that velocity fluid and the temperature fluid decreases as *Frank* – *Kamenetskii* parameter and variable viscosity parameter increases. A transient decrease in both the fluid velocity and temperature is observed with increase in each β , $\alpha_0 \& \alpha_3$ non-Newtonian parameters, E_1 Electric parameter, *Br* Brinkman number and Darcy number which decreases the porosity in the system of flow.

For engineering purpose, the flow model of our problem represents the oils well and as the ψ *Frank* – *Kamenetskii* parameter is increasing there is quick recovery of oil from the oils' well. Also, the results of this problem are of great interest in production processing, automobile engine, for the safety of life and proper handling of the materials during processing.

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