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Reacting Flow of Temperature-Dependent Variable Permeability Through a Porous Medium in the Presence of Arrhenius Reaction

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Abstract: In this work, a reacting flow of temperature-dependent variable permeability through a porous medium has studied and analyzed. It is assumed that the viscosity is temperature-dependent. Absolute permeability is said to be the ability of a porous medium to transmit its fluid content under an applied pressure gradient. The permeability of a porous medium is an intrinsic property that measures its ability to transmit fluids or a medium that allow fluids to pass through it. It is determined by the macroscopic properties of the medium namely porosity, pore size distribution, tortuosity and specific surface. It is a well known fact that the value of permeability is independent of the type of fluid used in the porous medium where Newtonian fluids are used for measurements. Since a porous medium contains pores, the applications of thermal science to the system leads to an intuitive believe that the thermal expansion coefficient of the solid component will cause volumetric expansion with changes in temperature variation of the medium. The governing partial differential equations are transformed into ordinary differential equations in terms of a suitable similarity variable. Galerkin weighted residual method is employed to solve the resulting non-linear equations and the effects of various physical parameter s involve in the system of flow were reported graphically. Furthermore, the effect of variable permeability parameter on the velocity profile is investigated. Some special cases with their physical significance are discussed and compared with the existing published work.

Keywords: Unsteady Gravity Flow, Weighted Residual Method, Power-Law Fluid and Viscous Dissipation

1. Introduction

In recent years, heat and mass transfer problems with chemical reaction are of importance in many practical processes such as distribution of temperature and moisture over agricultural field, energy transfer in a wet cooling tower, in the method of generating and extracting power from a moving fluid. So, many researchers have taken interest in studying the above mentioned effects.

Soil and earth dams that are made of filters are very much based upon the permeability of a saturated porous soil under load. Permeability is refers to as fractional part of the proportionality constant in Darcy's law. Modified Darcy's law relates the rate of flow and properties of the fluid to the pressure gradient that is applied to the saturated porous medium. This implies that permeability is an essential component of the porous medium. The standard procedure for determining the permeability of porous media is based on the fundamental assumption that as long as viscous flow prevails the absolute permeability of a porous medium is a property of the medium, and is it is independent of the fluid used. [1]

This important discovery may have significant ramifications in much oil recovery by thermal processes. The injection of hot water and steam into oil reservoirs, underground combustion, injection of fluids in to wells, the production of geothermal energy, and the disposal of atomic waste products i n porous formations all cause changes formation temperatures. In reservoir engineering, absolute permeability basic parameter which has often been measured at room conditions, with the implicit assumption that only confining pressure affected the result. However, a single value of absolute permeability throughout a range of reservoir temperatures has been used in reservoir engineering calculations. [2] Furthermore, the expansion of the solids and the matrix material of the porous medium will combine to n decrease the pore spaces available for fluid flow. Consequently, thermo mechanical effects in addition to the macroscopic properties of the porous medium can lead to permeability change. Also, parameter used in quantifying the ability of a fluid phase to flow in the porous medium in the presence of another is the effective permeability which is the permeability for a given saturation of the medium.

Nowadays, the offshore petroleum industry, oil recovery processes are continuously looking for modern technologies in order to enhance the optimal oil recovery and to optimize the operation. The main challenges encountered by the oil industry are low oil recovery factor, depletion of oil production, gas coning and water coning. Researchers have shown that considerable amount of oil still remain in the reservoir after well shutdown. The residual oil saturation in the porous medium and early breakthrough are the main reasons for remaining oil volumes in abandon oil fields. Moreover, since the properties of the reservoir have a considerable effect on the oil recovery processes, it is of great importance to identify the relation between the recovery factor and the properties of the reservoir. Consequently, the relative permeability of the fluid varies with time due to changing in water saturation during the recovery of oil processes. If relationships and variations of the change in permeability due to time are known, it implies that recovery the oil from the reservoir will be efficiently upgraded by manipulating properties of the reservoir. Enhanced Oil Recovery (EOR) methods can be used to change the properties such as relative permeability and residual oil saturation. A typical oil reservoir consists of an underlying aquifer and a gas cap. Heavy oil in the reservoirs occupy more than two third of the global oil reserves. The oil recovery factor is the ratio between the amount of oil that can be extracted from a reservoir throughout the lifespan of the well and the total reservoir oil in place. [3]

The high demand of oil and gas would remain until fan appreciable measure of a cost effective feasible energy source is found. Consequently, the oil industry will always look for modern techniques in order to extract as much oil and gas as possible in an efficient and effective method. Recovery processes have been developing for years in order to extract maximum output by increasing the recovery factor and controlling the water breakthrough. These two features are major strong functions of properties of the reservoir and extraction techniques. The regulated reservoir properties will expose the access to control the recovery factor and the coning effect of the oil in the reservoir. Non-Newtonian fluids flow through porous media is of interest for various engineering and industrial applications such as penetration of glue in the surface porosity of solid materials, injection of muds, slurries or cement grouts to reinforce soils, propagation of blood through kidney, injection of drilling fluids in rocks either for the reinforcement of the wells or for enhancing oil recovery, among others. [3-4]

It is a well known fact that the vast majority of materials expand on heating, which is an indication of a positive coefficient of thermal expansion. At the simplest level this phenomenon can be traced back to the asymmetric shape of a typical inter atomic potential well. For a simple diatomic molecule, the gradual population of higher energy vibration levels will lead to an increase in bond distance as temperature increases. A porous material containing the fluid is a non-homogeneous medium and there can be numerous in homogeneities that are present in a porous medium. However, the permeability of the porous medium is not negligible. In nature, so many flows exist which are caused not only by the temperature differences but also by concentration differences. These mass transfer differences do affect the heat transfer rate. Modified Darcy flow being highly viscous in nature, for most fluid of engineering application the viscosity is strongly dependent on the temperature. However, the thermal diffusivity remains relatively constant. [5-10]

The study of mass and heat transfer flow in saturated porous reservoirs has received much attention in the recent times due to its ever increasing applications in industries and in modern contemporary technology. Phenomena of transport in a saturated porous medium are encountered in many engineering disciplines. Internal temperature-dependent heat generation is primarily related to a practical problem such as the sterilization of liquid foods by continuous ohmic heating. Kumar and Prasad [11] considered MHD pulsatile flow through a porous medium. Analytical solution was employed in solving the system of flow. Their result shows that an increase in the permeability parameter and Hartmann number leads to a decrease in the steady state velocity. Vajraveh et al [12] examined fluid flow and heat transfer over a permeable stretching cylinder. A numerical method involving second order finite difference scheme known as Keller Box method was employed to investigate the velocity and temperature distribution of the flow system. Their result shows that increasing values of the fluid viscosity parameter is to enhance the temperature. This is due to the fact that an increase in the fluid viscosity parameter results in an increase in the thermal boundary layer thickness. The effects of variable viscosity, viscous dissipation and chemical reaction on heat and mass transfer flow of MHD micropolar fluid along a permeable stretching sheet was examined by Salem [13]. A numerical method involving Runge-Kuta fourth order method and shooting technique were employed to investigate the velocity and temperature distribution of the flow system. The results show that as Prandtl number and viscosity parameter increases the velocity profile and the temperature profile decreases. The effect of variable viscosity and thermal conductivity of micro polar fluid in a porous channel in the presence of magnetic field was studied by Gitima [14]. A numerical method involving Runge-Kuta fourth order method was employed to investigate the velocity and temperature distribution of the flow system. The results show that the velocity and temperature of the fluid increases as Darcy number, thermal conductivity variation parameter and magnetic field parameter increases. Bataller [15] investigated on unsteady gravity flows of a power-law fluid through a porous medium. [15] analyzed the flow in two direction, one side both thinning and thickening of the fluids and on the other hand, two different types of solutions, for the case of a gravity flow generated by the injection of a power-law fluid at the well into an empty reservoir of an infinite extent. He employed shooting method to analyze the flow model. The result shows that as power-law index increases the velocity profile decreases. Ogunsola and Ayeni [16] considered the effects of temperature distribution of an Arrheniusly reacting unsteady flow through a porous medium with variable permeability. A numerical method involving shooting method was employed to investigate the velocity and temperature distribution of the flow system. Their result shows that as Frank-Kamenetskii parameter increases the fluid velocity and temperature increases.

Motivated by these facts, the present work has been undertaken in order to analyze a reacting flow of temperature-dependent variable permeability through a porous medium by considering the effects of viscous dissipation on the flow system.

2. Mathematical Formulation

In this work, unsteady gravity flow with viscous dissipation of a power-law fluid through a porous medium is considered. The governing equations are continuity, momentum and energy equations. The governing equations are analyzed using a similarity transformation in terms of a similarity variable. Considering a two dimensional flow in the $x-z$ plane where the free surface is a streamline at a point on the surface, we expressed the flow by a modified Darcy's law.

$$\frac{1}{r}\frac{\partial(rhu)}{\partial r} = -\Phi\frac{\partial h}{\partial t} \tag{1}$$

Where $\Phi = g/2m, \Phi$ being the porosity of the porous medium, which is assumed to be constant in both space and time. u is given below as:

$$u = -\left(\frac{K(T)\rho}{\mu_{ef}}\right)^{\frac{1}{n}} \frac{\partial h}{\partial s}\left|\frac{\partial h}{\partial s}\right|^{\frac{1-n}{n}} \tag{2}$$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \mu\left(\frac{\partial u}{\partial r}\right)^2 + Q(T) \tag{3}$$

The appropriate initial and boundary conditions for this work are

$$h(0,t) = h_0, h(0,t) = h_1, h(\infty,t) = h_0, T(r,0) = T_0,\ T(0,t) = T_1,\ T(\infty,t) = T_0\quad t > 0 \tag{4}$$

where

ρ is the density, C_p -Specific heat at constant pressure, μ - Dynamic viscosity, $\mu\left(\frac{\partial h}{\partial r}\right)^2$ is the viscous dissipation, u - Component of velocity in the radial direction, μ_{ef} is the effective viscosity, n -dimensionless Power-law index, $k-$ thermal conductivity of the fluid, k_0 -thermal expansion exponent, T_0 -Initial temperature and it is the reference temperature, T -Temperature within the boundary layer, $T_1,\ T_2, ...T_\infty$ - Temperature at the plate, η -Apparent viscosity, i.e. Similarity variable parameter, f -is a dimensionless stream function, θ -Dimensionless temperature.

The first two terms on the right hand side of equation represent the heat conduction and reacting term.

3. Method of Solution

Following [15] let assume reaction term, variation of thermal conductivity, and permeability to be of the form:

$$Q(T) = {Q_0}/{t}\,(T - T_0),\ k(T) = k_0 e^{b(T-T_0)}, K(T) = K_0\left[1 - \Lambda_0(T - T_0)\right] \tag{5}$$

Taking the positive values of ${\partial h}/{\partial r}$ since the velocity which is expressed in terms of the fluid height in the medium is negative equations (1)-(5) yield.

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\left(\frac{\rho}{\mu_{ef}}\right)^{\frac{1}{n}}\left(rh\left(\left(\frac{\partial h}{\partial r}K_0\left[1-\Lambda_0(T-T_0)\right]\right)^{1/n}\right)\right)\right] = \Phi\frac{\partial h}{\partial t} \tag{6}$$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{k_0}{r}\frac{\partial}{\partial r}\left(e^{b(T-T_0)} r\frac{\partial T}{\partial r}\right) + \mu\left(\frac{\rho}{\mu_{ef}}\right)^{\frac{1}{n}}\left(\frac{\partial}{\partial r}\left(\frac{\partial h}{\partial r}K_0\left[1-\Lambda_0(T-T_0)\right]\right)^{1/n}\right)^2 + \frac{Q_0}{t}(T-T_0) \tag{7}$$

Introducing the following dimensionless variables

$$\theta = \frac{T - T_0}{T_1 - T_0}, T = \theta(T - T_0) + T_0,\ t' = \frac{t}{t_0}, r' = \frac{r}{R}, h' = \frac{h}{h_0} \tag{8}$$

For radial axisymetric flow

$$h(r,t)=h_1\, f(\eta);\eta=rt^{\beta},\theta(r,t)=t^{\lambda}\varphi(\eta),\, h_1=h_3t^{\alpha} \tag{9}$$

Equations (6)-(9) becomes

$$\left\{\eta(f')^{\frac{1}{n}}f'+(1-\Lambda\varphi)^{1/n}+(-\Lambda\varphi)^{1/n}\eta f(f')^{\frac{1}{n}}+(1-\Lambda\varphi)^{1/n}\frac{d}{d\eta}(f')^{\frac{1}{n}}\right\}=a^2\eta(\alpha f+\beta\eta f') \tag{10}$$

$$\Pr(\lambda\varphi+\beta\eta\varphi')=\frac{d}{d\eta}\left(e^{\gamma\varphi}\eta\varphi'\right)\Pr D\varphi-Ec\Pr\left[\frac{d}{d\eta}\left((1-\varphi\gamma)f'\right)^{\frac{1}{n}}\right]^2 \tag{11}$$

Dimensionless boundary conditions

$$f(0)=1, f(\infty)=0,\ \ ,\varphi(0)=1,\varphi(\infty)=0 \tag{12}$$

where

$$Ec=\frac{\mu R_4}{c_p(T_1-T_0)},\Pr=\frac{\rho c_p}{k_0},\gamma=b(T_1-T_0),\left[\frac{K_0\rho}{\mu}\Lambda_0(T_1-T_0)\right]^{2/n}=D,S^2=\left(\frac{\mu}{K_0\rho}\right)^{\frac{1}{n}}\Phi, \tag{13}$$

Ec is the Eckert number (due to viscous heating), $\Pr$ is the Prandtl number, γ is the variable thermal conductivity parameter, D is the reacting parameter or heat source, and S^2 is the porosity parameter.

A numerical solution of Equations (10)-(13) together with the boundary conditions using Galerkin-weighted residual method as follows:
let

$$f=\sum_{i=0}^{2}A_ie^{y},\varphi=\sum_{i=0}^{2}B_ie^{\left(-i/4\right)y} \tag{14}$$

A maple software was used to solve problem and the result are presented in Figures 1-6.

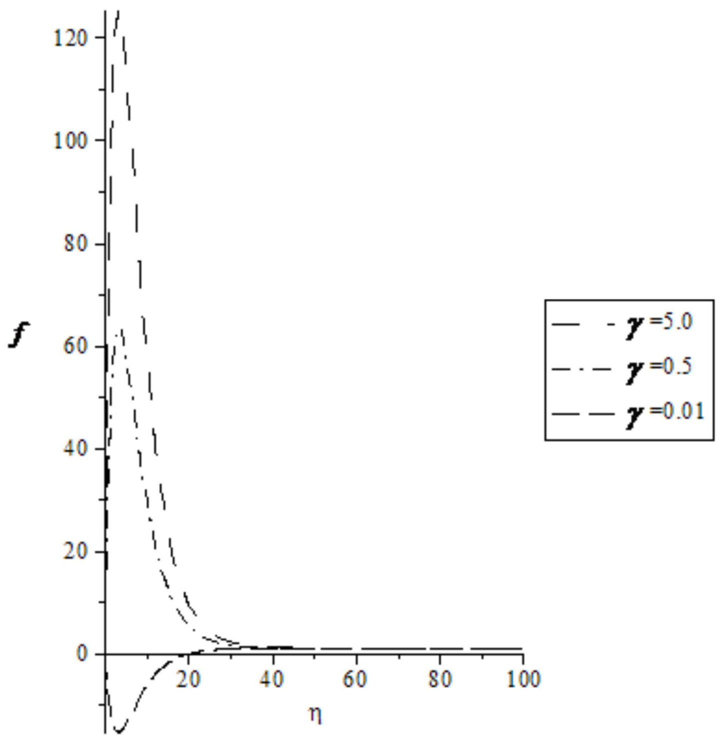

Figure 1. *Graph of the velocity function* f *for various values of* γ *when* $\beta=-0.001$,$\alpha=0.001,S=0.001,\Pr=\lambda=\Lambda=n=1.0,D=Ec=1.1$.

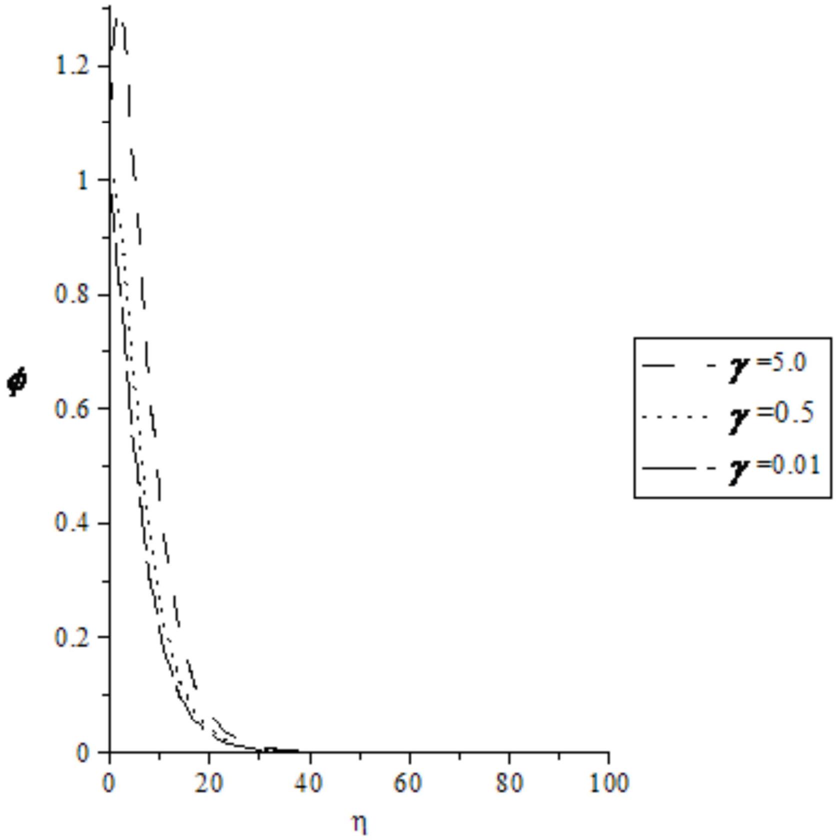

Figure 2. *Graph of the temperature function* φ *for various values of* γ *when* $\beta = -0.001$, $\alpha = 0.001, a = 0.001, \Pr = n = 1.0, \gamma = Ec = 1.1$.

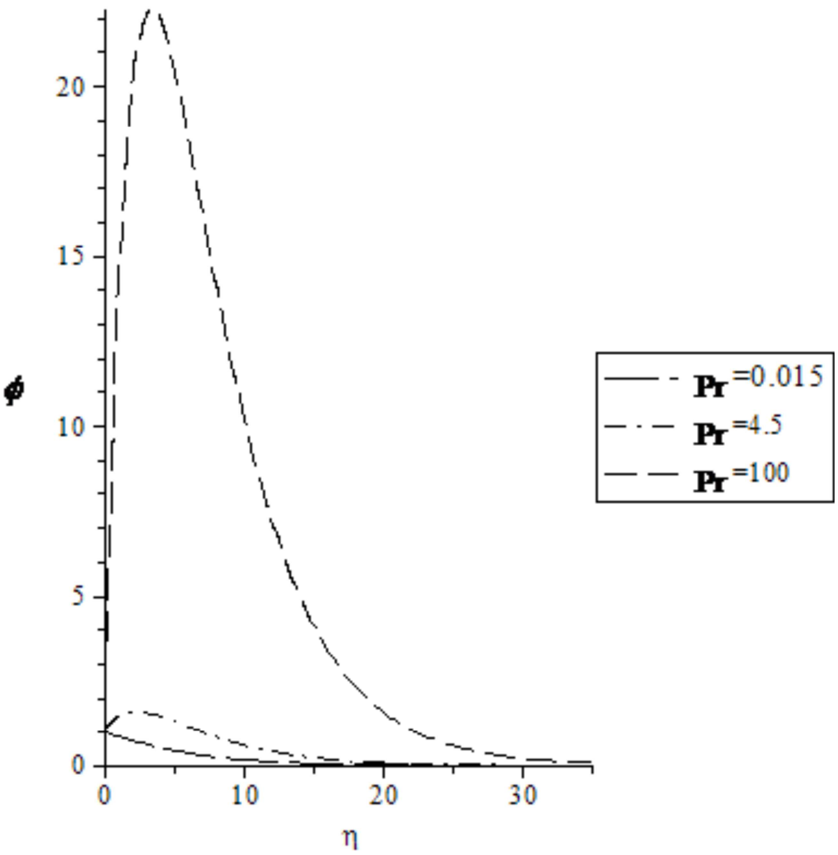

Figure 3. *Graph of the temperature function* φ *for various values of* Pr *when* $\beta = -0.001$, $\alpha = 0.001, a = 0.001, S = n = 1.0, D = Ec = 1.1$.

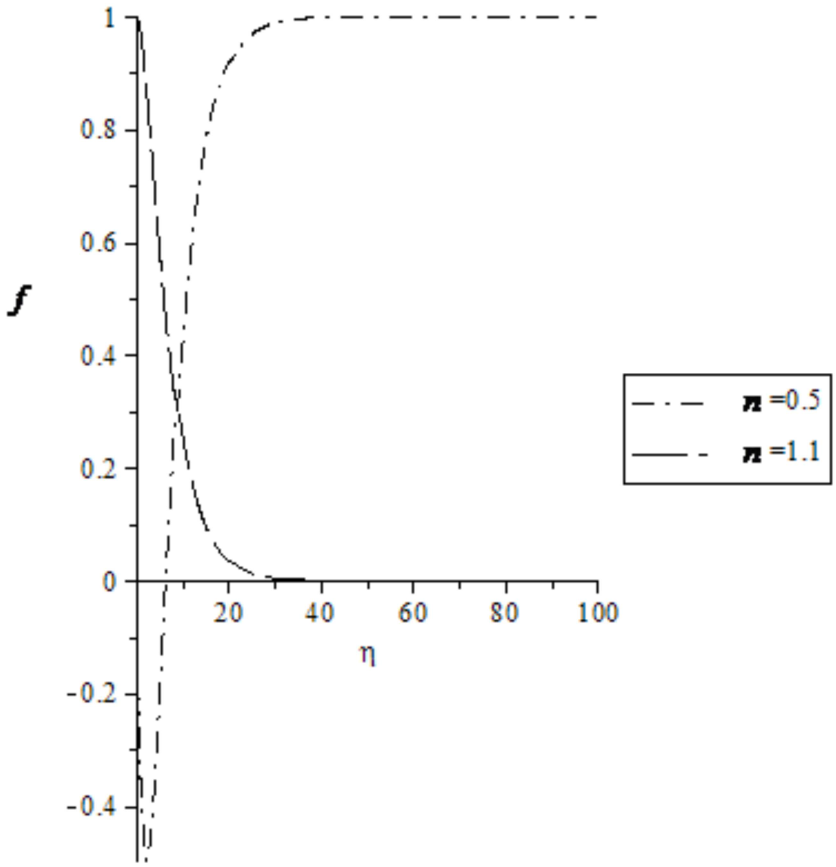

Figure 4. *Graph of the velocity function f for various values n when $\alpha = 0.5, a = \beta \geq 0.1$.*

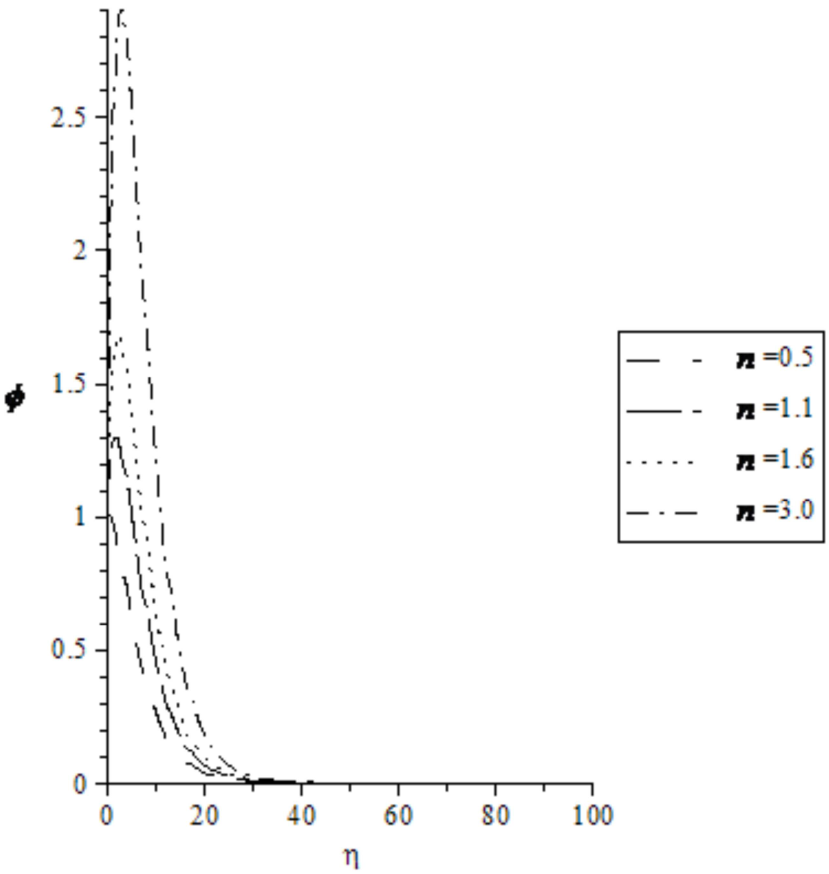

Figure 5. *Graph of the temperature function φ for various values n, when $\beta = -0.001$, $\alpha = 0.001, S = 0.001, \Pr = \lambda = \Lambda = \gamma = 1.0, D = Ec = 1.1$.*

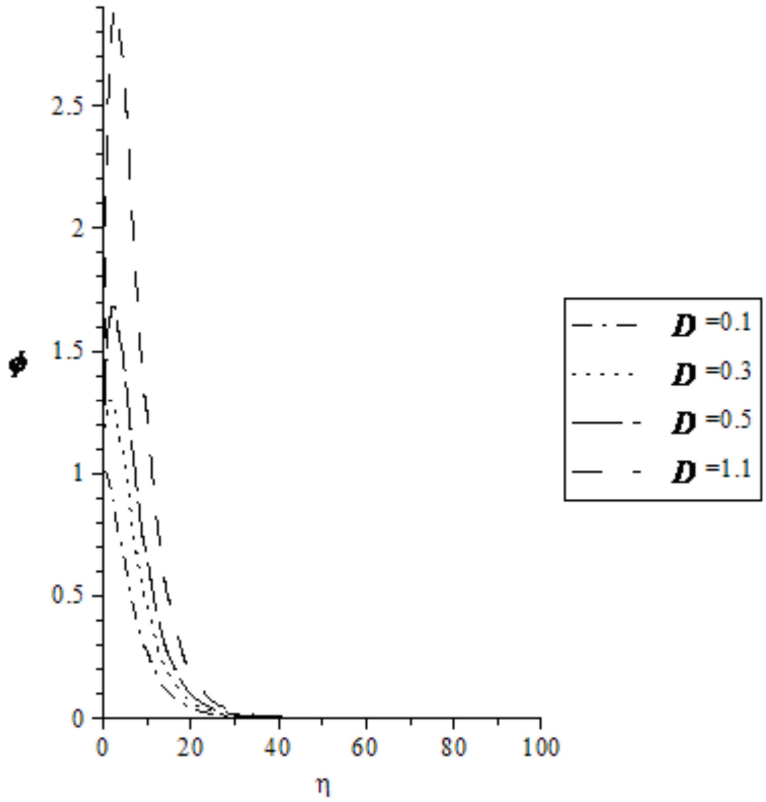

Figure 6. *Graph of the temperature function* φ *forvarious values of D , when* $\beta=-0.001$, $\alpha=0.001, S=0.001, \Pr=\lambda=\Lambda=\gamma=1.0, n=Ec=1.1$.

4. Discussion of Results

A suitable model for the reacting flow of temperature-dependent variable permeability through a porous medium has been considered. From figures 1-2 the result shows that the fluid velocity and the fluid temperature decrease as γ increases. It is noted that If γ is increased with small values, its effect on temperature behaviour may not be observed, hence the high values is employed in order to show the effect. Consequently, increase in the reacting fluid thermal conductivity together with a decrease in the fluid viscosity will boost the flow model. It is observed in figure 3 that the temperature is found to decrease with an increase in $\Pr$ Prandtl number. Low Prandtl number of a reacting fluid is more thermally stable than high Prandtl number and physically the fluid viscosity increases while the thermal diffusivity decreases, leading to accumulation of heat leading to an increasing viscous dissipation. It can be seen from figure 6 that as D permeability parameter increases the temperature of the fluid this is due to the effect of viscous dissipation. The physical effect is that temperature is enhanced effectively than when there is no porosity. Porosity and permeability of the oil reservoir rock are important factors to be considered in an oil recovery processes.

5. Conclusion

It is observed that velocity decreases as power-law index increases and parameter n divides into two when $n<1$ the fluid is a shear thinning and $n>1$ the fluid is shear thickening. It is also observed that as the reacting parameter increases it enhances the temperature which enhances quick recovery of oil from the reservoir. It is observed from figure 3 that as $\Pr$ increases the viscosity of the fluid increases and thereby decreasing the temperature. Physically, it implies that for smaller values of $\Pr$ the heat spread out quickly to the heated surface more rapidly compared to the momentum (velocity). It can be concluded that the increase physical parameters i.e. Eckert number, Prandtl number, and reacting parameter leads to a corresponding decrease in the viscosity of the fluid. These will be of great interest to the field engineers in various processes of oil recovery.

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