MIXED CONVECTIVE BOUNDARY LAYER SLIP FLOW OVER A VERTICAL POROUS PLATE THROUGH A POROUS MEDIUM

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Abstract

In this work, effects of slip at the boundary on the mixed convective boundary layer flow over a flat plate through a porous medium is considered. The governing partial differential equations were transformed into ordinary differential equations in terms of suitable similarity variable. We employed Galerkin weighted residual method to solve the resulting non-linear equations. The results show the effects of variable viscosity parameter, Brinkman number, Reynolds number, Prandtl number and Darcy number on the flow system.

Keywords: Mixed convection, boundary layer flow, Weighted residual method, vertical plate, and porous medium.

INTRODUCTION

The thermal buoyancy which is generated due to heating/cooling of a vertical porous plate has a large impact on flow and heat transfer characteristics. Combined forced and natural convection has been widely studied from both theoretical and experimental standpoint over the past few decades. Convection heat transfer in the fluid flows are phenomena of great interest from theoretical and practical angles because of its large area of applications in engineering practices and geophysical fields

Heat transfer problem of mixed convection has been studied by several authors: Barletta et al [1] considered dual mixed convection flows in a vertical channel. Ingham et al [2] studied combined free and forced convection in vertical channels of porous media. Olajuwon [3] examined the flow and natural convection heat transfer in a power-law fluid past a vertical plate with heat generation. The fundamental importance of convective flow in porous media has been established in the recent books by Nield and Bejan [4], Ingham et al.[5] Bejan and Kraus [6]. The above studies of free and mixed convection flow in vertical channels are based on the hypothesis that the fluids are non-Newtonian. Moreover, because of their technological importance, studies involving free, forced and mixed convection flow of non-Newtonians in channels are very important in several industrial processes. Szeri and Rajagopal [7] examined the flow of a Non-Newtonian fluid between heated parallel plates. Howarth [8] numerically considered various aspect of the Blasius flat-plate flow problem. Sparrow and Cess [9] the effect of magnetic field on free convection heat transfer on isothermal vertical plate. Krishnendu et al [10] considered similarity solution of mixed convective boundary layer slip flow over a vertical plate. Motivated by the work of Krishnendu et al [10], we considered variable viscosity and non-constant thermal conductivity of a mixed convective boundary layer slip flow over a vertical porous plate through a porous medium.

GOVERNING EQUATIONS AND METHOD OF SOLUTION

Following Krishnendu et al [10], we consider the steady two-dimensional laminar mixed convective flow of a viscous incompressible fluid over a vertical plate. Using boundary layer approximation, the basic governing equations are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right)+\rho g\beta^{*}\left(T_1-T_\infty\right)+\mu_{ef}\,\frac{u}{K}=0$$

$$\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(k(T)\frac{\partial u}{\partial y}\right)+\frac{\mu(T)}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\mu_{ef}\frac{u^2}{\rho c_p K}=0 \quad (2)$$

The appropriate initial and boundary condition are as follows

$$u(0)=L_1\left(\frac{\partial u}{\partial y}\right), v=0, u\to U_\infty, T(y)=T_w+D_1\left(\frac{\partial T}{\partial y}\right),\ as\ y=0,\ T\to T_\infty\ as\ y\to\infty$$

where θ -is the dimensionless temperature, k is the permeability of the porous media, K is the thermal conductivity, ρ is the density, C_p is the specific heat at constant pressure, μ is the dynamic viscosity, $\mu\left(\frac{\partial u}{\partial r}\right)^2$ is the viscous heating effect, direction, ψ is the stream function, η is the similarity variable, μ_{ef} is the effective viscosity, e^T is the thermal expansion, T_0 is the fluid initial temperature or wall temperature, T_r is the reference temperature, T is the absolute temperature within the boundary layer, $T_1, T_2, ...T_\infty$ - Temperature at the plate, β^* is the volumetric coefficient of thermal expansion, g is the acceleration due to gravity, u is the dimensionless velocity, y is the dimensionless transversal coordinate, T_∞ is the free stream temperature, $L_1=L(\mathrm{Re}_x)^{1/2}$ is the velocity slip factor and $D_1=D(\mathrm{Re}_x)^{1/2}$ is the thermal slip factor with $L\, and\, D$ being the initial values of velocity and thermal slip factors having same dimension of length and $\mathrm{Re}_x=U_\infty x/\upsilon, U_\infty$ is the free stream velocity, $T_\infty=T_\infty+T_0/x$ is the variable temperature increase along the plate.

From Equations (2) and (3) we seek variable thermal conductivity and Reynolds model of the form

$$k(T)=k_0e^{-\alpha\theta}, \mu(T)=\mu_0e^{-M\theta} \quad (5)$$

$$\frac{\partial}{\partial y}\left(\mu_0e^{-M\theta}\frac{\partial u}{\partial y}\right)+\rho g\beta^*(T_1-T_\infty)+\mu_{ef}\frac{u}{K}=0$$

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$$\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(k_0 e^{-\alpha\theta}\frac{\partial u}{\partial y}\right)+\frac{\mu_0 e^{-M\theta}}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\mu_{ef}\frac{u^2}{\rho c_p K}=0$$

we introduce the following variables and parameters

$$\psi=\sqrt{U_\infty \upsilon x}\, f(\eta), T=T_\infty+\left(T_w-T_\infty\right)\theta(\eta) \text{ and } \eta=y\sqrt{U_\infty/\upsilon x}, u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x},$$

$$y'=\frac{y}{lo}, u'=\frac{u}{u_0}, \theta=\frac{T-T_{wr}}{T_1-T_w}, Da=\frac{k}{Ulo^2}.$$

Substituting (4)&(5) into (1) & (2) and dropping the primes we obtain

$$\frac{d}{d\eta}\left(e^{-M\theta}f''\right)+1/2\, ff''+\lambda\theta-\frac{u}{Da}=0$$

$$\frac{1}{\Pr}\frac{d}{d\eta}\left(e^{-\alpha\theta}\theta'\right)+\frac{Br}{Pe}e^{-M\theta}\left(u'\right)^2+\frac{1}{2}f\theta'+f'\theta+\frac{u^2}{Da}=0$$

where

$$\lambda=\frac{lo^2\rho g\beta\left(T_1-T_\infty\right)}{\mu_0 u_0}, \frac{Br}{Pe}=\frac{\mu_0 u_0 lo^2}{lo^2\rho c_p\left(T_1-T_\infty\right)lo}, \Pr=\frac{lo\rho c_p\mu_{ef}}{k_0}$$

The transformed boundary condition are as follows

$$f(0)=0, f(0)=\delta q(0), f(\infty)=1, \theta(0)=1+\beta z(0), \theta(0)=0, \theta(\infty)=0$$

We proceed to solve equations (7) and (8) subject to (9) numerically using Galerkin-Weighted Residual Method as follows:

let $f=\sum_{i=0}^{2}A_i e^{y}, \theta=\sum_{i=0}^{2}B_i e^{\left(-i/4\right)y}$ (10)

The results are presented in Figures 1-6

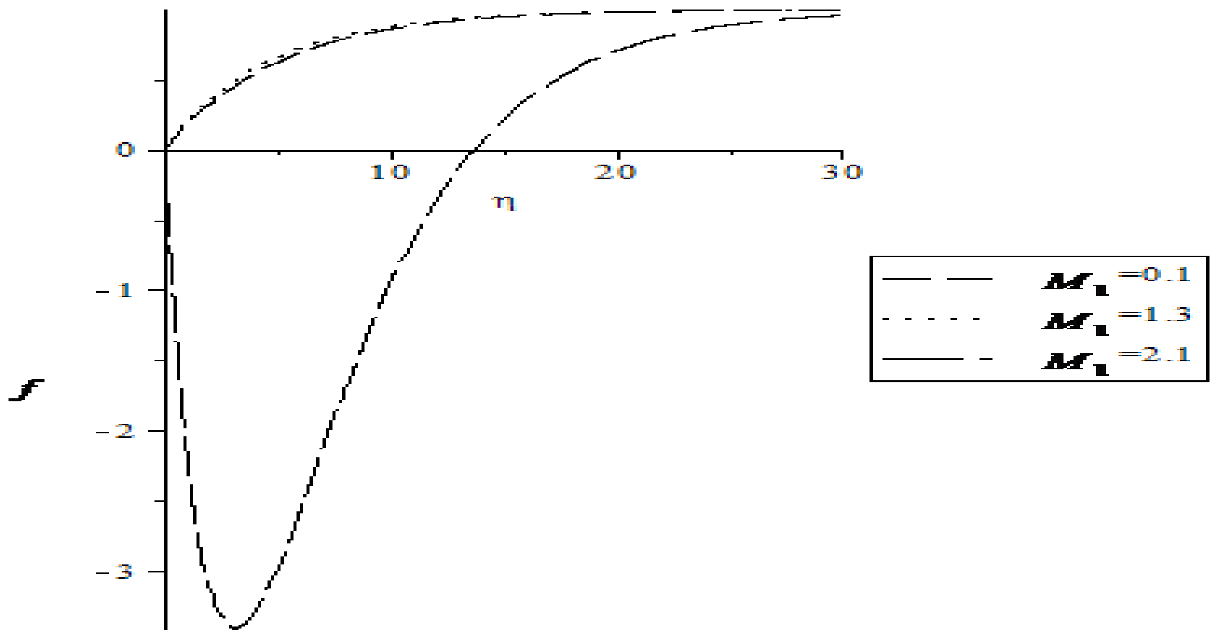

Figure1: Graph of the velocity function u for various values of $Br = 0.5, \Pr = \lambda = 1.0$

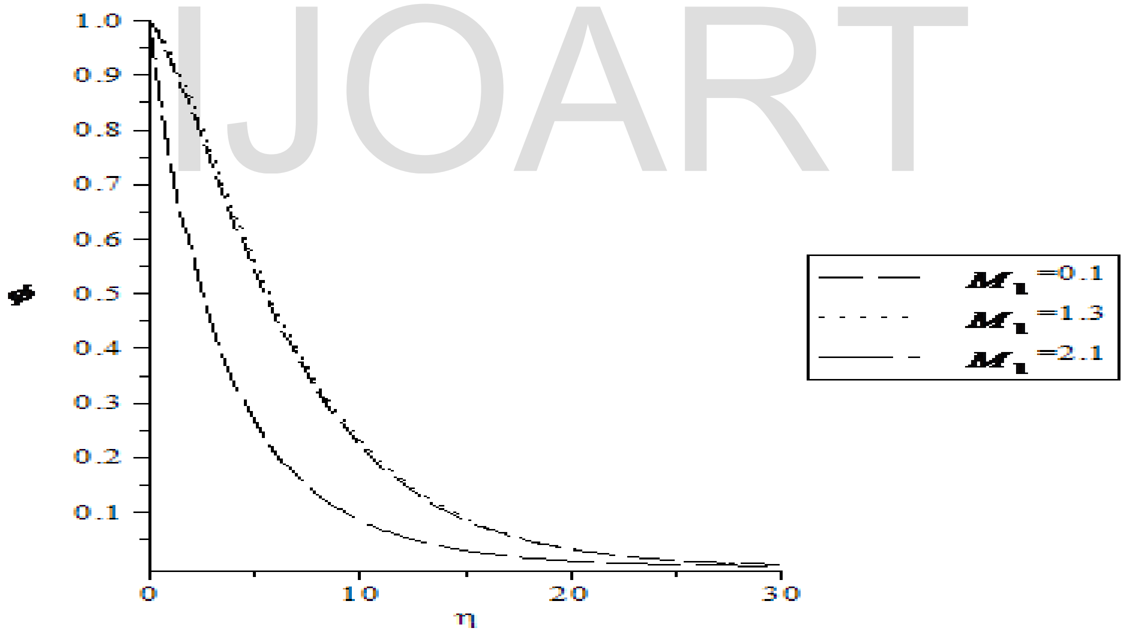

Figure 2: Graph of the temperature function θ for various values of $Br = \Pr = Gr = 1.0, Pe = 0.75$

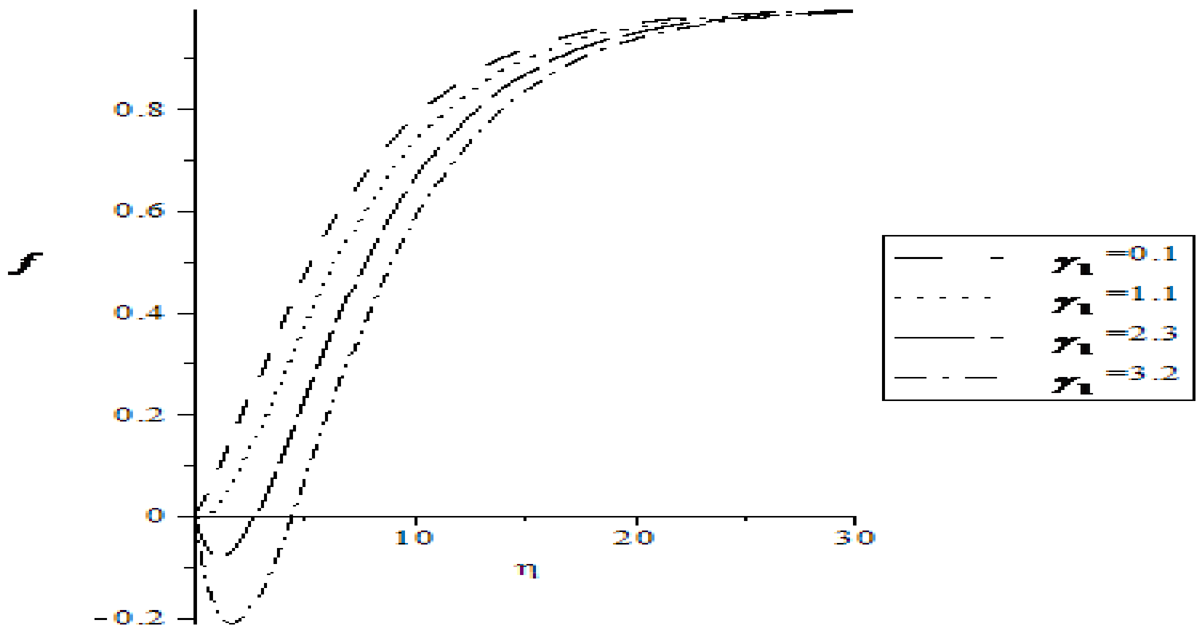

Figure3: Graph of the velocity function u for various values of $Br = 0.5, \Pr = Gr = 1.0$

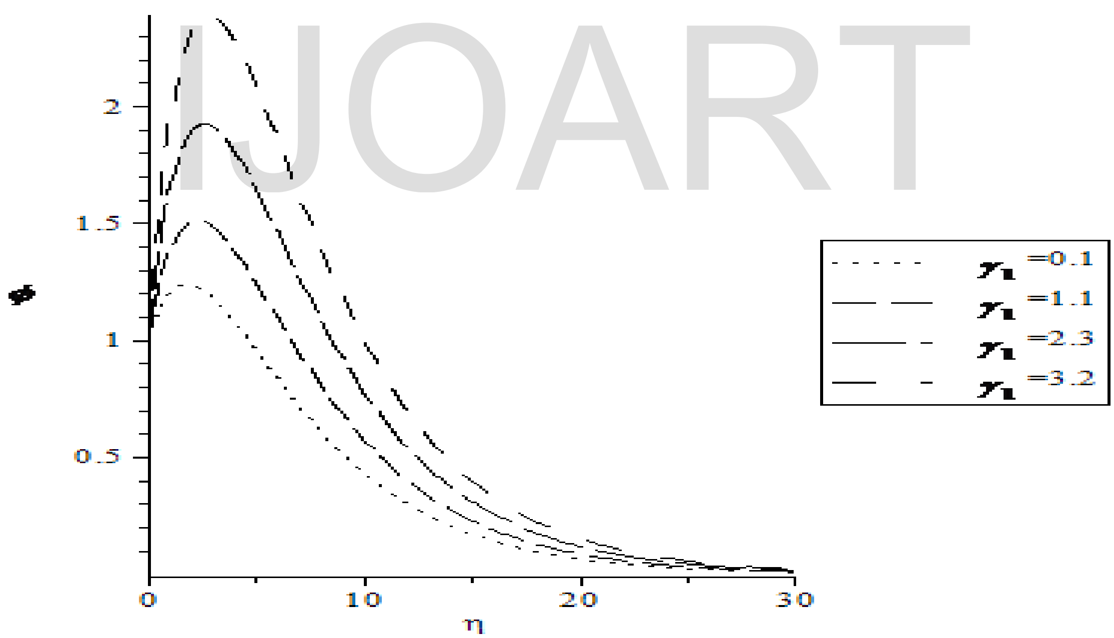

Figure 4: Graph of the temperature function θ for various values of $Br = \Pr = Gr = 1.0, Pe = 0.75$

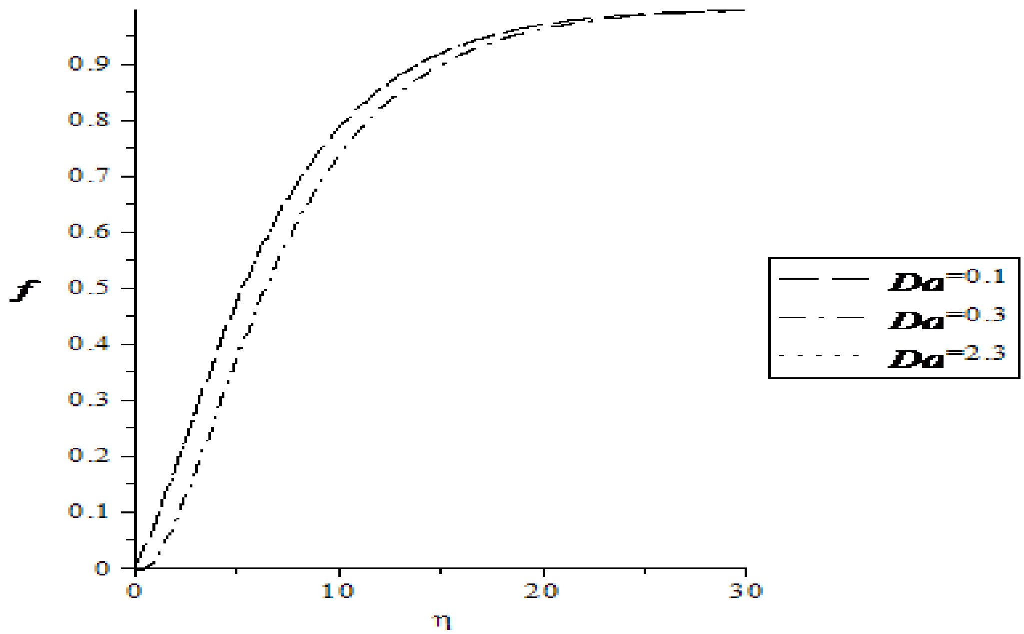

Figure5: Graph of the velocity function u for various values of $Br = 0.5, \Pr = \lambda = 1.0$

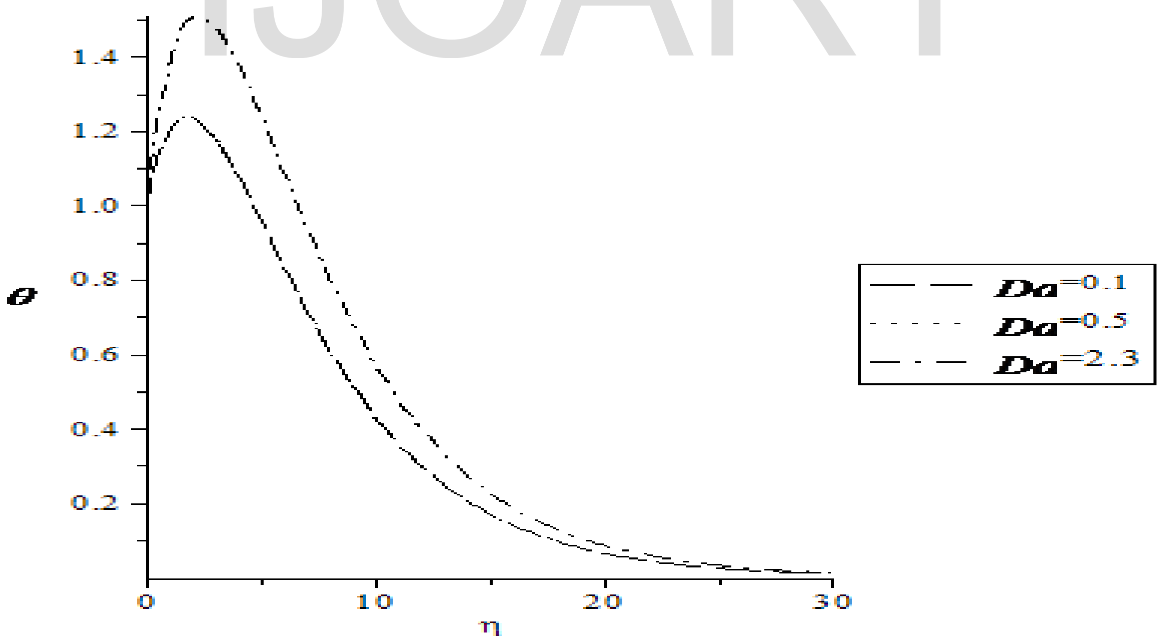

Figure 6: Graph of the temperature function θ for various values of $Br = \Pr = Gr = 1.0, Pe = 0.75$

Discussion of Results/Conclusion

From Figures1,3 &5 the results show that the velocity field increases as $Da, \mathrm{Re}, M, Pe, Br$ parameters, and γ_1 thermal conductivity parameter increases. From Figures 2 ,4 &6the result shows that the temperature field decreases with increase in each $Da, \mathrm{Re}, M, Pe, Br$ parameters, and γ_1 thermal conductivity parameter.

Conclusion

It can be concluded that increase in the physical parameters lead to an increase fluid velocity and decrease in the fluid temperature. For engineering purpose, the results of this problem are of great interest in oil recovery processes, for the safety of life and proper handling of the materials during processing.

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